# CALCULUS
## DERIVATIVES AND LIMITS

## DERIVATIVE DEFINITION

$$\frac{d}{dx}(f(x)) = f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

## BASIC PROPERTIES

$$(cf(x))' = c(f'(x))$$
$$(f(x) \pm g(x))' = f'(x) \pm g'(x)$$
$$\frac{d}{dx}(c) = 0$$

## MEAN VALUE THEOREM

If $f$ $is$ differentiable on the interval $(a, b)$ and continuous at the end points there exists a $c$ in $(a, b)$ such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

## PRODUCT RULE

$$(f(x)g(x))' = f(x)'g(x) + f(x)g(x)'$$

## QUOTIENT RULE

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$$

## POWER RULE

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

## CHAIN RULE

$$\frac{d}{dx}\left(f(g(x))\right) = f'(g(x))g'(x)$$

## COMMON DERIVATIVES

$$\frac{d}{dx}(x) = 1$$
$$\frac{d}{dx}(\sin x) = \cos x$$
$$\frac{d}{dx}(\cos x) = -\sin x$$
$$\frac{d}{dx}(\tan x) = \sec^2 x$$
$$\frac{d}{dx}(\sec x) = \sec x \tan x$$
$$\frac{d}{dx}(\csc x) = -\csc x \cot x$$
$$\frac{d}{dx}(\cot x) = -\csc^2 x$$
$$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1 - x^2}}$$
$$\frac{d}{dx}(\cos^{-1} x) = -\frac{1}{\sqrt{1 - x^2}}$$
$$\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1 + x^2}$$
$$\frac{d}{dx}(a^x) = a^x \ln(a)$$
$$\frac{d}{dx}(e^x) = e^x$$
$$\frac{d}{dx}(\ln(x)) = \frac{1}{x}, x > 0$$
$$\frac{d}{dx}(\ln|x|) = \frac{1}{x}$$
$$\frac{d}{dx}(\log_a(x)) = \frac{1}{x \ln(a)}$$

## CHAIN RULE AND OTHER EXAMPLES

$$\frac{d}{dx}([f(x)]^n) = n[f(x)]^{n-1}f'(x)$$
$$\frac{d}{dx}\left(e^{f(x)}\right) = f'(x)e^{f(x)}$$
$$\frac{d}{dx}(\ln[f(x)]) = \frac{f'(x)}{f(x)}$$
$$\frac{d}{dx}(\sin[f(x)]) = f'(x)\cos[f(x)]$$
$$\frac{d}{dx}(\cos[f(x)]) = -f'(x)\sin[f(x)]$$
$$\frac{d}{dx}(\tan[f(x)]) = f'(x)\sec^2[f(x)]$$
$$\frac{d}{dx}(\sec[f(x)]) = f'(x)\sec[f(x)]\tan[f(x)]$$
$$\frac{d}{dx}(\tan^{-1}[f(x)]) = \frac{f'(x)}{1 + [f(x)]^2}$$
$$\frac{d}{dx}\left(f(x)^{g(x)}\right) = f(x)^{g(x)}\left(\frac{g(x)f'(x)}{f(x)} + \ln(f(x))\, g'(x)\right)$$

## PROPERTIES OF LIMITS

These properties require that the limit of $f(x)$ and $g(x)$ exist

$$\lim_{x \to a}[cf(x)] = c \lim_{x \to a} f(x)$$
$$\lim_{x \to a}[f(x) \pm g(x)] = \lim_{x \to a} f(x) \pm \lim_{x \to a} g(x)$$
$$\lim_{x \to a}[f(x)g(x)] = \lim_{x \to a} f(x) \lim_{x \to a} g(x)$$
$$\lim_{x \to a}\left[\frac{f(x)}{g(x)}\right] = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)} \text{ if } \lim_{x \to a} g(x) \neq 0$$
$$\lim_{x \to a}[f(x)]^n = \left[\lim_{x \to a} f(x)\right]^n$$

## LIMIT EVALUATION METHOD – FACTOR AND CANCEL

$$\lim_{x \to -3} \frac{x^2 - x - 12}{x^2 + 3x} = \lim_{x \to -3} \frac{(x+3)(x-4)}{x(x+3)} = \lim_{x \to -3} \frac{(x-4)}{x} = \frac{7}{3}$$

## L'HOPITAL'S RULE

If $\lim_{x \to a} \dfrac{f(x)}{g(x)} = \dfrac{0}{0}$ or $\dfrac{\pm\infty}{\pm\infty}$ then $\lim_{x \to a} \dfrac{f(x)}{g(x)} = \lim_{x \to a} \dfrac{f'(x)}{g'(x)}$

## LIMIT EVALUATIONS AT +-∞

$$\lim_{x \to \infty} e^x = \infty \text{ and } \lim_{x \to -\infty} e^x = 0$$
$$\lim_{x \to \infty} \ln(x) = \infty \text{ and } \lim_{x \to 0^+} \ln(x) = -\infty$$
If $r > 0$ then $\lim_{x \to \infty} \dfrac{c}{x^r} = 0$

If $r > 0$ & $x^r$ is real for $x < 0$ then $\lim_{x \to -\infty} \dfrac{c}{x^r} = 0$

$$\lim_{x \to \pm\infty} x^r = \infty \text{ for even } r$$
$$\lim_{x \to \infty} x^r = \infty \ \& \ \lim_{x \to -\infty} x^r = -\infty \ \text{ for odd } r$$